



UNIVERSIDAD DE CHILE  
FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS  
DEPARTAMENTO DE FÍSICA

FI7007- Waves called Fronts

**Curso general 15 UD)**

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**Requisitos:**

FI3101 Mecánica Clásica

FI4004 Electrodinámica

Introducción

Los sistemas compuestos por muchos constituyentes microscópicos fuera del equilibrio termodinámico exhiben una gran variedad de comportamientos colectivos. Uno de los objetivos centrales de la *Física No-lineal* es la caracterización de estos comportamientos colectivos.

El gran éxito de esta área de la Física en las dos últimas décadas no solo en las ciencias naturales como la Física, Química y la Biología, sino también en las ciencias humanas tales como la Sociología y la Economía, se debe a que su objetivo principal es *el estudio de fenómenos robustos*, es decir, fenómenos independientes de la Física subyacente, por ejemplo, fenómenos ondulatorios, comportamientos caóticos, bifurcaciones, inestabilidades y formación de patrones.

Ondas no lineales que conectan diferentes estados de equilibrio, frentes, en sistemas disipativos extendidos han sido observado en diferentes campos, tales como: materiales magnéticos, cristales líquidos, corrientes filamentos en la descarga de gas, reacciones químicas, superficie del fluido, medios granulares, convección térmica, ondas solitarias en óptica no lineal, entre otros. Por lo tanto, se puede inferir la universalidad de Frentes.

Objetivos Principales

El objetivo principal de este curso es responder a las siguientes preguntas:

- Caracterización de las ondas no lineales tipo frentes en sistemas fuera del equilibrio.

- Caracterización de diferentes tipos de frentes.
- Determinación de la velocidad y forma de Frentes.
- Caracterización de bifurcaciones exhibidas por los frentes. Aplicación a fluidos, reacciones químicas, medios granulares, cristales líquidos, medios excitables, medios activos (cavidad óptica), etc.
- Interacción entre frentes.

## Evaluación (Curso general 15 UD)

Tareas bisemanales. El examen consistirá en un seminario sobre el análisis de un artículo o manuscrito de investigación.

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